

MRI-PHY-96-21

hep-ph/9607227

.....

**Implications of m_t and R_b on $Zt\bar{t}$
couplings in standard ETC models**

Uma Mahanta

Mehta Research Institute

10 Kasturba Gandhi Marg

Allahabad-211002, India

Abstract

In standard ETC models the sideways and diagonal ETC interactions contribute to δR_b with opposite signs. The aim of this article is to study the implications of the CDF value for m_t and the LEP value for R_b on $zt\bar{t}$ couplings where the LH sideways and diagonal ETC effects interfere constructively. We find that for $m_t = 175$ GeV, $\delta R_b = .0022$ and $m_s^2 = m_d^2$, F_v^t and F_a^t are modified by 19% and 7% respectively from their SM values. The constraints implied by these deviations on diagonal ETC scenarios and the feasibility of probing them at NLC through polarization and angular distribution studies in $e^+e^- \rightarrow t\bar{t}$ are also considered.

In standard ETC models the large mass ($m_t \approx 175$ Gev) of the top quark is presumably due to sideways ETC dynamics (that connect ordinary fermions to technifermions) at relatively low energy scales (≈ 1 Tev) [1]. Because of $SU(2)_L$ gauge invariance of ETC interactions the same sideways ETC dynamics also gives rise to a sizeable neagtive shift (1.8%) in R_b [2] that can be detected with the present LEP precision [3] in measuring R_b ($R_b^{exp} \approx .2178 \pm .0011$). On the other hand diagonal ETC interactions (between a pair of technifermions or a pair of ordinary fermions) give rise to a positive correction to R_b [4]. The overall contribution to δR_b can therefore be of either sign and it can be large or small depending upon the relative size of the sideways and diagonal contributions. In contrast the sideways and diagonal ETC interactions interfere constructively in δg_l^t . Hence it is possible for a low enough ETC scale, that both contributions to δg_l^b are individually quite large in magnitude but their difference is small so as to fit the observed δR_b which is at the level of a few percent only. Such a scenario would produce large deviations from the SM in the LH $zt\bar{t}$ couplings. The aim of this article is threefold: i) to investigate if the recent experimental values of m_t and R_b imply large corrections to g_l^t and g_r^t in standard ETC models ii) the constraints imposed by these deviations on the unknown parameters of the model and iii) the feasibility of probing the deviations at NLC.

To illustrate our point we shall consider the one family TC model of Appelquist and Terning [5]. For simplicity the TF's will be assumed to be in the fundamental representation of an $SU(N)_{TC}$ gauge group. It can be shown that in this model the sideways ETC gauge boson exchange gives rise to the following four-fermion Lagrangian [4]

$$\begin{aligned}
L_{4f}^s = & -\frac{(g_{E,L})^2}{2m_s^2} \bar{Q}_L \gamma^\mu \psi_L \bar{\psi}_L \gamma_\mu Q_L - \frac{(g_{E,R}^U)^2}{2m_s^2} \bar{U}_R \gamma^\mu t_R \bar{t}_R \gamma_\mu U_R \\
& - \frac{(g_{E,R}^D)^2}{2m_s^2} \bar{D}_R \gamma^\mu b_R \bar{b}_R \gamma_\mu D_R.
\end{aligned} \tag{1}$$

On the other hand the diagonal ETC gauge boson gives rise to the four fermion Lagrangian

$$L_{4f}^d = \frac{1}{4m_d^2(N_{TC} + 1)}(g_{E,R}^U - g_{E,R}^D)\bar{Q}_R\gamma^\mu\tau_3 Q_R(g_{E,L}\bar{\psi}_L\gamma_\mu\psi_L + g_{E,R}^U\bar{t}_R\gamma_\mu t_R + g_{E,R}^D\bar{b}_R\gamma_\mu b_R). \quad (2)$$

Here $g_{E,L}$ is the effective ETC gauge coupling to LH fermions. $g_{E,R}^U$ ($g_{E,R}^D$) is the effective ETC gauge coupling to RH fermions with $I_3 = 1/2$ ($I_3 = -1/2$). We shall assume that the techniquark sector is intrinsically isospin symmetric i.e. $\langle\bar{U}U\rangle = \langle\bar{D}D\rangle$. To obtain the large mass splitting between t and b under this condition requires that $g_{E,R}^U \gg g_{E,R}^D$. Since spontaneous CSB in the TC sector occurs only in the I=1 channel, from L_{4f}^d we have dropped those terms which contain isospin singlet TF current. Fierz transforming the above expression for L_{4f}^s both with respect to Dirac and gauge group indices and dropping terms which contain isospin singlet TF current we get

$$L_{4f}^s = -\frac{(g_{E,L})^2}{4m_s^2 N_c}\bar{Q}_L\gamma^\mu\tau_a Q_L\bar{\psi}_L\gamma_\mu\tau_a\psi_L - \frac{(g_{E,R}^U)^2}{4m_s^2 N_c}\bar{Q}_R\gamma^\mu\tau_3 Q_R\bar{t}_R\gamma_\mu t_R + \frac{(g_{E,R}^D)^2}{4m_s^2 N_c}\bar{Q}_R\gamma^\mu\tau_3 Q_R\bar{b}_R\gamma_\mu b_R. \quad (3)$$

Below the TC chiral symmetry breaking scale we must replace the TF current by the appropriate sigma model current [6]. Considering only the term involving the weak Z boson we get in unitary gauge

$$\bar{Q}_L\gamma^\mu\tau_3 \otimes 1_3 Q_L = i\frac{f_Q^2}{2}Tr(\Sigma^+\tau_3 \otimes 1_3 D^\mu\Sigma)_{\Sigma=1} = -\frac{g}{2c}N_c f_Q^2 Z^\mu. \quad (4a)$$

$$\bar{Q}_R\gamma^\mu\tau_3 \otimes 1_3 Q_R = i\frac{f_Q^2}{2}Tr(\Sigma\tau_3 \otimes 1_3 (D^\mu\Sigma)^+)_{\Sigma=1} = \frac{g}{2c}N_c f_Q^2 Z^\mu. \quad (4b)$$

where 1_3 is the unit operator in color space. The sideways ETC induced non-standard couplings of t and b to Z boson are therefore given by

$$L_{4f}^s = \frac{(g_{E,L})^2}{8m_s^2} \frac{g}{c} f_Q^2 Z_\mu \bar{\psi}_L \gamma^\mu \tau_3 \psi_L - \frac{(g_{E,R}^U)^2}{8m_s^2} \frac{g}{c} f_Q^2 Z_\mu \bar{t}_R \gamma^\mu t_R$$

$$+ \frac{(g_{E,R}^D)^2}{8m_s^2} \frac{g}{c} f_Q^2 Z_\mu \bar{b}_R \gamma^\mu b_R. \quad (5)$$

The above Lagrangian implies that

$$\delta g_L^{ts} \approx -\frac{(g_{E,L})^2 f_Q^2}{8m_s^2}, \delta g_R^{ts} \approx \frac{(g_{E,R}^U)^2 f_Q^2}{8m_s^2}. \quad (6a)$$

$$\delta g_L^{bs} \approx \frac{(g_{E,L})^2 f_Q^2}{8m_s^2}, \delta g_R^{bs} \approx -\frac{(g_{E,R}^D)^2 f_Q^2}{8m_s^2}. \quad (6b)$$

Similarly for the diagonal ETC exchange we obtain the following deviations from the SM couplings to Z boson

$$L_{4f}^d = \frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} \frac{g}{c} N_c f_Q^2 Z^\mu (g_{E,L} \bar{\psi}_L \gamma_\mu \psi_L + g_{E,R}^U \bar{t}_R \gamma_\mu t_R + g_{E,R}^D \bar{b}_R \gamma_\mu b_R). \quad (7)$$

Hence

$$\delta g_L^{td} \approx -\frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,L} N_c f_Q^2, \delta g_R^{td} \approx -\frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,R}^U N_c f_Q^2. \quad (7a)$$

$$\delta g_L^{bd} \approx -\frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,L} N_c f_Q^2, \delta g_R^{bd} \approx -\frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,R}^D N_c f_Q^2. \quad (7b)$$

Adding the sideways and diagonal contributions separately for the LH and RH couplings of t we get $\delta g_L^t \approx -\frac{(g_{E,L})^2 f_Q^2}{8m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,L} N_c f_Q^2$ and $\delta g_R^t \approx \frac{(g_{E,R}^U)^2 f_Q^2}{8m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,R}^U N_c f_Q^2$. Similarly for b we get $\delta g_L^b \approx \frac{(g_{E,L})^2 f_Q^2}{8m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,L} N_c f_Q^2$ and $\delta g_R^b \approx -\frac{(g_{E,R}^D)^2 f_Q^2}{8m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)}{8m_d^2(N_{TC} + 1)} g_{E,R}^D N_c f_Q^2$. We shall assume that $N_{TC} = 2$ so that the TC contribution to the S parameter is in agreement with the experimental bounds [4]. From δg_L^t and δg_R^t we can compute the non-standard vector and axial vector couplings of the top quark to Z boson

$$\begin{aligned}\delta g_v^t &\approx -\frac{(g_{E,L})^2 f_Q^2}{16m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)(g_{E,L} + g_{E,R}^U) f_Q^2}{16m_d^2} \\ &\quad + \frac{(g_{E,R}^U)^2 f_Q^2}{16m_s^2}.\end{aligned}\tag{9a}$$

$$\begin{aligned}\delta g_a^t &\approx \frac{(g_{E,L})^2 f_Q^2}{16m_s^2} + \frac{(g_{E,R}^U - g_{E,R}^D)(g_{E,L} - g_{E,R}^U) f_Q^2}{16m_d^2} \\ &\quad + \frac{(g_{E,R}^U)^2 f_Q^2}{16m_s^2}.\end{aligned}\tag{9b}$$

The non-standard contributions to the vector and axial vector form factors for $Zt\bar{t}$ vertex are therefore given by

$$\begin{aligned}\delta F_v^t &\approx \frac{1}{(g_v^t)_{sm}} \left[-\frac{(g_{E,L})^2 f_Q^2}{16m_s^2} - \frac{(g_{E,R}^U - g_{E,R}^D)(g_{E,L} + g_{E,R}^U) f_Q^2}{16m_d^2} \right. \\ &\quad \left. + \frac{(g_{E,R}^U)^2 f_Q^2}{16m_s^2} \right].\end{aligned}\tag{10a}$$

$$\begin{aligned}\delta F_a^t &\approx \frac{1}{(g_a^t)_{sm}} \left[\frac{(g_{E,L})^2 f_Q^2}{16m_s^2} + \frac{(g_{E,R}^U - g_{E,R}^D)(g_{E,L} - g_{E,R}^U) f_Q^2}{16m_d^2} \right. \\ &\quad \left. + \frac{(g_{E,R}^U)^2 f_Q^2}{16m_s^2} \right].\end{aligned}\tag{10b}$$

where $(g_v^t)_{sm} = \frac{1}{4} - \frac{2}{3}s^2$ and $(g_a^t)_{sm} = -\frac{1}{4}$. F_v^t and F_a^t are normalized to unity at tree level in the SM. δg_l^b and δg_r^b affects the precision EW measurements at the Z pole through Γ_b or R_b . However, since $\delta\Gamma_b \propto (g_l^b)_{sm}\delta g_l^b + (g_r^b)_{sm}\delta g_r^b$ and $|(g_r^b)_{sm}| \ll |(g_l^b)_{sm}|$, we can ignore the effect of δg_r^b on R_b . In other words precision measurements of R_b constrains only δg_l^b satisfactorily but leaves δg_r^b largely unconstrained. We shall therefore use the LEP value of δR_b to impose constrain on the ETC contributions to δg_l^b only. Since the sideways ETC gauge boson also contributes to m_t , another constrain on the

ETC gauge coupling and the sideways gauge boson mass will arise from the CDF ($m_t = 176 \pm 8 \pm 10 \text{ GeV}$) or D0 ($m_t = 199 \pm 20 \pm 22 \text{ GeV}$) value for m_t [7]. Using naive dimensional analysis [8] and large N_{TC} scaling we can write

$$m_t \approx -\frac{g_{E,L} g_{E,R}^U \langle \bar{U}_L U_R \rangle}{2m_s^2} \approx \frac{g_{E,L} g_{E,R}^U}{2m_s^2} 4\pi f_Q^3 \sqrt{\frac{N_c}{N_{TC}}}. \quad (11a)$$

The above eqn. implies that $g_{E,L}$ and $g_{E,R}^U$ must be of the same sign. For $m_t \approx 175 \text{ GeV}$, $\sqrt{3}f_Q \approx 247 \text{ GeV}$ and $N_{TC} = 2$ we get $\frac{g_{E,L} g_{E,R}^U f_Q^2}{m_s^2} \approx .1594$. On the other hand the LEP value of δR_b imposes the following constrain on the expression for δg_l^b in the limit of vanishing $g_{E,R}^D$ (in this limit δg_r^b vanishes)

$$\frac{(g_{E,L})^2 f_Q^2}{m_s^2} \approx .1594 \frac{m_s^2}{m_d^2} - 10.3361 \delta R_b. \quad (11b)$$

From (11a) and (11b) we obtain the relation $\frac{g_{E,L}}{g_{E,R}^U} \approx \frac{m_s^2}{m_d^2} - 64.8438 \delta R_b$. For given values of δR_b and m_t , the ETC contributions to δF_v^t and δF_a^t therefore depend only on the unknown parameter $\frac{m_s^2}{m_d^2}$. Here we shall consider only those values of $\frac{m_s^2}{m_d^2}$ which lie between .5 and 2. Since the LEP value for R_b has been changing continuously, we shall treat δR_b as an almost free parameter. More precisely we shall calculate δF_v^t and δF_a^t for $m_t = 175 \text{ GeV}$ and $\delta R_b = .0011, .0022 \text{ and } .0044$. Note that the difference between the most recent LEP value of R_b and R_b^{sm} is .0022. We find that for $\delta R_b = .0011$, δF_v^t (δF_a^t) are given by .024(-.084), -.202(-.077), -.465(-.136) for $\frac{m_s^2}{m_d^2} = .5, 1, \text{ and } 2$ respectively. On the other hand if $\delta R_b = .0022$, δF_v^t (δF_a^t) are given by .056(-.090), -.194(-.074) and -.460(-.133) for the same set of values of $\frac{m_s^2}{m_d^2}$. Finally for $\delta R_b = .0044$, δF_v^t (δF_a^t) are given by .169(-.126), -.179(-.068) and -.448(-.125).

We observe the following features in the ETC contributions to δF_v^t and δF_a^t .

i) In δF_v^t , the LH sideways contribution and the diagonal contributions (both LH and RH) appear with the same sign (negative). The RH sideways contribution however appears with opposite sign (positive) relative to the former. On the contrary in δF_a^t the

sideways contributions (both LH and RH) and the LH diagonal contribution appear with the same sign (negative). The RH diagonal contribution however appears with opposite sign (positive) giving rise to some amount of cancellation.

ii) δF_v^t is more sensitive to low scale ETC physics than δF_a^t primarily because $|(g_v^t)_{sm}| < |(g_a^t)_{sm}|$.

iii) For a given $\frac{m_s^2}{m_d^2} \geq 1$, as we increase δR_b both δF_v^t and δF_a^t decrease in magnitude. But the change is not that significant. On the other hand for $\frac{m_s^2}{m_d^2} < 1$, δF_v^t increases quite rapidly with increasing δR_b . However $|\delta F_a^t|$ increases only slightly under this condition. The reason being for $\frac{m_s^2}{m_d^2} \geq 1$ the terms that contribute constructively in δF_v^t dominate and they are not much sensitive to δR_b . On the other hand for $\frac{m_s^2}{m_d^2} < 1$, the term that contributes destructively in δF_v^t dominates and it is quite sensitive to δR_b .

iv) For a fixed δR_b , as we increase $\frac{m_s^2}{m_d^2}$ both δF_v^t and δF_a^t increase in magnitude. The effect is significant for both, but it is more dramatic for δF_v^t . This happens because with decreasing m_d^2 the diagonal contribution to δR_b increases. To get the same δR_b , the LH sideways contribution must therefore increase in magnitude and the two effects interfere constructively in δF_v^t and δF_a^t .

v) ETC interactions renormalize the $zt\bar{t}$ vertex in such a way that the strength of axial charge always decreases. On the other hand the magnitude of the vector charge decreases (increases) if $\frac{m_s^2}{m_d^2} \geq 1$ ($\frac{m_s^2}{m_d^2} \leq .5$) for all relevant values of δR_b .

Note first that QCD and EW corrections to F_v^t and F_a^t in the context of the SM are only of the order of a few percent or less. Thus large corrections ($\geq 10\%$) to these form factors would imply the presence of new physics. Second we find that even if δR_b is constrained to a few percent, the resulting δF_v^t and δF_a^t can be greater than 10% in standard ETC models particularly if $m_d^2 \leq m_s^2$. The main reason being in δR_b the sideways and diagonal ETC effects interfere destructively but in δF_v^t and δF_a^t they interfere constructively thereby giving a large effect [9].

The anomalous vector and axial vector couplings of the top quark to the Z boson

can be probed with high precision at NLC by studying the angular distribution of different polarization states of $t\bar{t}$ pair. Barklow and Schmidt [10] performed a tree level study of NLC sensitivity to these couplings by applying a maximum-likelihood analysis and using all the information (helicity angles) in $t\bar{t}$ event. The top mass was set to $m_t = 175$ GeV and the NLC parameters were chosen to be $\sqrt{s} = 400$ GeV, an integrated luminosity of 100 fb^{-1} and 90% polarization for electrons. The full maximum-likelihood analysis at 95% confidence level yields an error of 10% in F_v^t and F_a^t . The ETC induced corrections to F_v^t discussed in this article are therefore expected to be within the sensitivity reach of NLC for most of the natural values of the parameters. In addition δF_a^t will also be measurable with the projected NLC sensitivity provided $m_d^2 < m_s^2$.

From our study we can conclude that once δR_b is measured quite accurately at LEP, precision measurements of δF_v^t and δF_a^t at NLC can be used to put strong constraints on the ratio $\frac{m_s^2}{m_d^2}$. For example in order that δF_v^t and δF_a^t are less than the projected NLC precision of .100 for measuring them, it is clear that $\frac{m_s^2}{m_d^2}$ and δR_b must be less than 1 and .0022 respectively. On the other hand both δF_v^t and δF_a^t can exceed the 10% precision limit of NLC if $\frac{m_s^2}{m_d^2} \geq 2$ and $0 \leq \delta R_b \leq .0044$ or $\frac{m_s^2}{m_d^2} \leq .5$ and $\delta R_b \geq .0044$. Note however that $\frac{m_s^2}{m_d^2}$ cannot be much smaller than 1 for otherwise $\frac{g_{E,L}^U}{g_{E,R}^U}$ will become too small or negative. In any case the fact that the SM has been extremely successful in explaining almost all the collider data so far to a few percent implies that $\frac{m_s^2}{m_d^2} \geq 1$ is likely to be excluded by precision studies of $zt\bar{t}$ couplings at NLC.

It is important to compare the constraints on diagonal ETC scenarios arising from $zt\bar{t}$ vertex correction with those from $\delta\rho_{new}$. For $g_{E,R}^D = 0$ the ETC induced isospin violating four TF Lagrangian is given by $L_{\delta\rho}^{ETC} = -\frac{1}{4N(N+1)}\frac{(g_{E,R}^U)^2}{m_d^2}\bar{Q}_R\gamma^\mu T_3 Q_R\bar{Q}_R\gamma_\mu T_3 Q_R$. It then follows [4] that $\delta\rho_{new} = \frac{1}{N}\frac{(g_{E,R}^U)^2}{16m_d^2}f_Q^2$. For $\delta R_b = .0011$ and $\frac{m_s^2}{m_d^2} = .5, 1$ and 2 $\delta\rho_{new}$ is given by .0029, .0027 and .0026. On the other hand for $\delta R_b = .0022(.0044)$ $\delta\rho_{new}$ is given by .0035 (.0058), .0030 (.0035) and .0027 (.0029) for the same set of values of $\frac{m_s^2}{m_d^2}$. We find that for a fixed value of δR_b ($\frac{m_s^2}{m_d^2}$) $\delta\rho_{new}$ decreases (increases) as $\frac{m_s^2}{m_d^2}$ (δR_b) increases. The

present experimental bound [11] on $\delta\rho_{new}$ is $\delta\rho_{new} \leq .0040$. This implies that in order to satisfy the $\delta\rho_{new}$ constraint δR_b must be less than .0022 and $\frac{m_s^2}{m_d^2} \geq .5$ or $\frac{m_s^2}{m_d^2}$ must be greater than 1 and $0 \leq \delta R_b \leq .0044$. Comparing the constraints arising from $zt\bar{t}$ vertex correction with those from $\delta\rho_{new}$ we find that small ($< 10\%$) $zt\bar{t}$ vertex correction and small ($< .0040$) $\delta\rho_{new}$ can arise simultaneously in diagonal ETC scenario only if $\delta R_b < .0022$ and $\frac{m_s^2}{m_d^2} < 1$. It is clear therefore that diagonal ETC scenarios suffer both from large $zt\bar{t}$ vertex correction and large $\delta\rho_{new}$ problem for most values of δR_b and $\frac{m_s^2}{m_d^2}$. It has recently been shown [12] that the most dangerous weak-isospin violating effects in realistic commuting ETC models arise not from diagonal (TC singlet) ETC gauge bosons but from massive ETC gauge bosons in the adjoint representation of TC. The contribution of these gauge bosons to $\delta\rho_{new}$ is of order 6% which exceeds the present experimental bound by more than an order of magnitude. In order to solve the $\delta\rho_{new}$ problem in such models, either one has to fine tune the relevant ETC gauge coupling close to criticality or construct models that do not contain massive adjoint ETC gauge bosons.

Acknowledgement: The author would like to thank Dr. Sekhar Chivukula for careful reading of the preliminary version of the manuscript and for making useful comments.

References

1. R. S. Chivukula, K. Lane and A. G. Cohen, Nucl. Phys. B 343, 554 (1990); T. Appelquist, J. Terning and L. C. R. Wijewardhana, Phys. Rev. D 44, 871 (1991).
2. R. S. Chivukula, S. B. Selipsky and E. H. Simmons, Phys. Rev. Lett. 69, 575 (1992).
3. Talk presented by A. Blondel at the Warsaw ICHEP, July 1996.
4. N. Kitazawa, Phys. Lett. B 313, 395 (1995); G. H. Wu, Phys. Rev. Lett. 74, 4137 (1995); K. Hagiwara and N. Kitazawa, Phys. Rev. D 52, 5374 (1995).
5. T. Appelquist and J. Terning, Phys. Lett. B 315, 139 (1995).
6. H. Georgi, Weak Interactions and Modern Particle Theory, Benjamin-Cummings, Menlo Park, CA (1984).
7. F. Abe et. al. (CDF collaboration), Phys. Rev. Lett. 74, 2626 (1995); S. Abachi et.

- al. (DO collaboration), Phys. Rev. Lett. 74, 2632 (1995).
8. A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).
 9. The general idea that the sideways and diagonal ETC contributions interfere constructively in $zt\bar{t}$ vertex was first pointed out by Michael Peskin (unpublished).
 10. T. L. Barklow and C. R. Schmidt, in DPF 94: The Albuquerque Meeting, S. Seidel, ed. (World Scientific, Singapore, 1995); G. A. Ladinsky and C. P. Yuan, Phys. Rev. D 49, 4415 (1994).
 11. J. Erler and P. Langacker, Phys. Rev. D 52, 441 (1995).
 12. T. Appelquist, N. Evans and S. B. Selipsky, Phys. Lett. B 374, 145 (1996).